

New Proposed Approach for solving Assignment Problem and A Comparative study with Hungarian Method

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Abstract: Assignment Problem in fact is a special Case of transportation problem. In this paper I attempt to introduce a new approach for solving assignment problems. I examined numerical example by applying the new approach and compare the result with that of Hungarian method. The new approach is easy to apply for solving assignment problem.

Keywords: Assignment Problem, transportation problem, Hungarian method.

1. INTRODUCTION

Assignment problems are special type of allocation problems. This is particularly important in the theory of decision making. Some applications of the assignment model are the marriage problem, Machine Set-Up problems, machine installation problem etc. In a normal Case of assignment problem the objective is to assign the available resources to the activity going on so as to minimize cost or maximize total benefits. In this paper I proposed a simple approach for solving assignment problem and the proposed approach is supported by numerical examples.

Mathematical Formulation:-Suppose there are n-jobs which are to be performed by n-persons, each of whom can perform each of the Jobs with varying of efficiency. Suppose i th person is assigned to job J_i . The problem then is to find a permutation (J_1, J_2-J_n) from the set of $n!$ permutation such that the total assignment cost is minimum. Thus the cost matrix associated with the problem is given as under.

Table-I

		J_1	J_2	J_n	Availastility
Resources	1				1
	2				1
	n				1
Requirement		1	1		

Alternatively the problem can be for restated as follows:

Let $x_{ij} = \begin{cases} 1 & \text{if } j\text{th job assigned to } i\text{th person} \\ 0 & \text{if not} \end{cases}$

Then problem can be stated as

$$\text{Min. } z = \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij}$$

$$\text{s.t } \sum_{j=1}^n x_{ij} = 1, i=1,2,\dots,n$$

$$\sum_{i=1}^n x_{ij} = 1, j=1,2,\dots,n$$

$$x_{ij} = 0 \text{ or } 1 \text{ for all } i \text{ and } j$$

Proposed approach:

In this section a simple and practical new approach is proposed for solving balanced / unbalanced assignment problem. And supported by numerical example. The result is then compared with that of Hungarian method.

The Proposed Approach Having Following Steps:

Step I:

Examine the cost matrix. And identify the least cost cell.

Step II: Subtract the least cost element from all other elements. While doing so there is atleast one zero in the reduced matrix. Make the as assignment where zero occurs. If there are more zero in the reduced matrix, Then make the assignment accordingly.

Step III: Now drop the row and corresponding column in which assignment is made. Again identify the least cost Cell in the sub matrix thus obtained.Repeat the step II to get an assignment schedule.

2. NUMERICAL COMPARISON OF PROPOSED APPROACH WITH HUNGARIAN METHOD

Example-I A Company has 4 Machines on which 4-jobs are to be preformed. Each Job can be assigned to one and only one Machine. Find optimum assignment.

		Machine			
		I	II	III	IV
Job	I	2	5	5	4
	II	8	6	7	9
	III	4	5	8	7
	IV	6	7	6	5

Step I: The least cost cell is c_{11} with cost 2.

Step II: Subtract 2 from where matrix

		Machine			
		I	II	III	IV
I	<input type="checkbox"/> 0	3	3	3	2
II	6	4	5	5	7
III	2	3	6	6	5
IV	4	5	4	4	3

There is one zero in first row and first column so make the assignment.

Step III: Drop first row and first column to get a sub matrix.

$$B = \begin{bmatrix} 4 & 7 & 7 \\ 3 & 6 & 5 \\ 5 & 4 & 3 \end{bmatrix}$$

Repeat step I and step II to get the reduce sub matrix.

$$B = \begin{matrix} & & \text{I} & \text{II} & \text{III} \\ \text{II} & & 1 & 2 & 4 \\ \text{III} & & 0 & 3 & 2 \\ \text{IV} & & 2 & 1 & 0 \end{matrix}$$

Now there are two zero in the reduced matrix. So make the assignments. Apply step III to matrix B.

III

Let $C = II [2]$ Now make assignment is this cell.

Refor optimum assignment is

$$I - II - III - II - IV - IV$$

And minimum set up time is $2+7+5+5=19$

The optimum assignment schedule when solved by Hungarian method is

	I	II	III	IV
I	0	2	1	1
II	3	0	0	3
III	0	0	2	2
IV	2	2	0	0

And minimum set up time is 19

Example II: Consider unbalanced assignment problem. Find optimum assignment schedule for the problem.

	1	2	3
I	9	26	15
II	13	27	6
III	35	20	15
IV	18	30	20

(In the proposed approach there is no need to balance it.)

Step I: Least element in the matrix 6

Step II: Subtract 6 from each element to get a matrix

	1	2	3
I	3	20	3
II	7	21	0
III	29	14	9
IV	12	24	14

Make assignment in the cell (2, 3)

Let the sub matrix (After dropping 2nd row and 3rd column)

$$B = \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{cc} 1 & 2 \\ \hline \text{I} & \begin{array}{cc} 3 & 20 \end{array} \\ \text{III} & \begin{array}{cc} 29 & 14 \end{array} \\ \text{IV} & \begin{array}{cc} 12 & 24 \end{array} \end{array} \quad \text{Men}$$

Apply step I and II

Reduced Matrix is

$$B = \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{cc} 1 & 2 \\ \hline \text{I} & \begin{array}{cc} 0 & 17 \end{array} \\ \text{III} & \begin{array}{cc} 26 & 11 \end{array} \\ \text{IV} & \begin{array}{cc} 9 & 21 \end{array} \end{array}$$

Apply step III and get a new submatrix

$$C = \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{c} 2 \text{ MEN} \\ \left(\begin{array}{c} 11 \\ 21 \end{array} \right) \\ \text{III} \\ \text{Tosu} \\ \text{IV} \end{array}$$

Apply step I & II C =

$$\begin{array}{c} \\ \\ \\ \end{array} \begin{array}{c} 2 \text{ MEN} \\ \left(\begin{array}{c} 0 \\ 10 \end{array} \right) \\ \text{III} \\ \text{IV} \end{array}$$

Thus the assignment schedule is

I - I - II - 3 III - 2 and IV -- 4

The minimum time is $9 + 6 + 20 = 35$

The optimum assignment schedule given by Hungarian method is

	I	II	III	IV
I	0	6	9	0
II	4	7	0	0
III	26	0	9	0
IV	9	10	14	0

And minimum Cost is 35

A new approach for solving assignment problem is introduced. Two example (one balanced / problem and one unbalanced problem) using proposed approach and Hungarian method are examined and optimal solutions are compared among two methods and optimal solutions by both methods are same.

Table 2: Comparison of optimal values by two methods

Example	Hungarian method	Proposed Approach	Optimum Value
I	19	19	19
II	35	35	35

There for the proposed approach is simple and easy to apply for solving assignment problem.

3. CONCLUSION

In this paper I proposed a new approach for solving assignment problem. The proposed approach which is different from the existing methods is supported by numerical examples. The optimal solution in both examples is same as given by Hungarian method. Therefore this paper introduces a different approach which is simple and easy to solve assignment problem.

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